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Wave Transformation using Unit Delayed Reflection

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Abstract **– Wave variables have been used to overcome the transmission delay of a system from the viewpoint of passivity. During implementation of the wave transformation for the sampled-data system, a unit delay arises due to causality of the reflection wave. The property of the wave transformation using delayed reflection, which is different from the standard wave transformation, can be verified with the passivity condition of the scattering matrix. The unit delay on the reflection wave occasionally becomes a positive damping. From the passivity condition of the sampled-data system, the wave impedance can be designed such that the wave transformation using delayed reflection wave provides the effect of positive damping. As an example, the wave transformation using delayed reflection wave makes it possible that the stable haptic interface is achieved on the sampled-data system with appropriate wave impedance. The simulation and the experiment results show that the stable haptic interface can be accomplished by use of the wave transformation using delayed reflection.**

Index Terms - Wave transfomation; Sampled-data system; Delayed reflection; Haptic interface.

I. Introduction

When a system is represented in terms of power variables (i.e., force and velocity), time delays such as transmission delay or unit delay caused by discretization tend to degrade the system performance. As an example, the transmission delay in the bilateral teleoperation can make the system unstable [1]. Therefore, the wave space composed of wave variables have been widely used to deal with time delay problems in the bilateral teleoperation [2][3][4]. The wave transformation on the sampled-data system inevitably has a unit delay corresponding to the sampling period of the system. This delay is required for the causality due to the reflection wave which is the feedback wave used to compute the other wave variable. Therefore, this unit delay makes the wave transformation on the sampled-data system different from the standard wave transformation on the continuous-time system. For the systems sensitive to the length of the sampling period such as the haptic interface, this unit delay affects stability and transparency of the system, but this delay associated with the wave transformation on the sampled-data system has not been considered seriously so far. The wave impedance that determines the behavior of a system is an essential parameter in the wave transformation. In application of the wave transformation for the sampled-data system, it is desirable that the wave impedance be designed such that the unit delayed reflection wave represents a passive element.

Anderson and Spong suggested a delayed transmission line that was designed for bilateral teleoperation based on the passivity and scattering theory. In the transmission line, the norm of the scattering matrix was one, and there was no loss [1]. Niemeyer and Slotine showed that the time delay was not an active element in the wave space composed of wave variables [2]. The wave variables stemmed from the scattering parameter which is the ratio of the reflected power wave to the incident power wave [5]. The wave variables were mainly used to deal with time delay for bilateral teleoperation [3][4].

Colgate and Schenkel found that the active virtual wall could make the haptic interface unstable and presented the theoretical passivity condition for the haptic interface implemented on the sampled-data system [6]. In their research, they suggested that an increase in damping of a haptic device and sampling frequency could improve stability of the haptic interface. Hannaford and Ryu suggested the passivity controller (PC) based on the passivity observer (PO) in the time domain [7]. In their PO/PC scheme, the PO monitored the passivity level of the system, and generated damping just enough to make the system passive. Most researches on haptic interfaces have focused on designing the fixed or varying virtual coupling [8][9].

Since the wave transformation implemented on the sampled-data system inevitably involve the unit delayed reflection wave, this will be referred to as the wave transformation using the unit delayed wave reflection or simply the *Wave Transformation using Delayed Reflection* (WTDR). Furthermore, the transmission through the wave transformation will be referred to the wave transmission. In view of passivity, the property of the wave transmission through WTDR can be analyzed by the scattering matrix which is a function of the sampling period and frequency. It can be shown that the norm of the scattering matrix is bounded depending on the choice of the sampling period at all frequencies. Furthermore, the unit delayed reflection line space can serve as a positive damping element in some frequency range. Therefore, a system performing the wave transmission through WTDR can satisfy the passivity condition with an appropriate sampling period. Remarkably, the wave transmission through WTDR is robust to not only the transmission delay but also the unit delay that makes the system unstable.

A haptic interface usually has several active elements such as time delay caused by numerical integration and feedback signal, and sampling and quantization related to the discrete-time system. Due to these active elements, the haptic interface on the sampled-data system sometimes exhibits unstable behavior especially when a large amount of power is involved (e.g., contact with the very stiff wall). From the passivity condition of the sampled-data system, the wave transmission through WTDR is occasionally a passive element (i.e., positive damping) depending on the choice of the sampling period and wave impedance. The objective of this paper is to provide the guideline for the design of the wave impedance that makes the haptic interface stable by considering the passivity condition of the sampled-data system. The experiment results showed that the operation of the haptic interface based on the proposed wave transmission through WTDR could generate satisfactory performance on the virtual environment.

This paper is organized as follows: Section 2 introduces the wave variables, and Section 3 deals with the wave transformation using delayed reflection implemented on the sampled-data system and the property of the scattering matrix. As an example, the wave transmission through WTDR is applied to the basic haptic interface in Section 4. In Section 5, the stable haptic interface is extended to a haptic device and experimental results are presented. Finally, conclusions are drawn in Section 6.

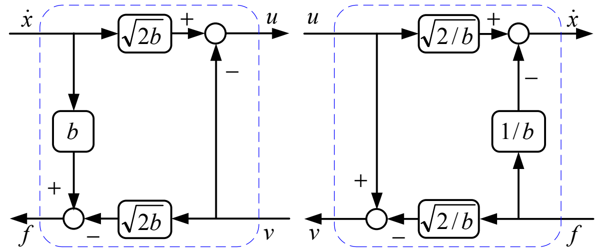
II. Wave Transformation

Power is usually described by the power variables such as force and velocity. It can be also represented by the wave variables which will be defined below. The conversion between power variables and wave variables can be performed by the wave transformation. Figure 1 illustrates two types of wave transformation. The configuration of Fig. 1(a) is called the *impedance type* wave transformation because the velocity input causes the force output. Likewise, the configuration of Fig. 1(b) is called the *admittance type* wave transformation. It is easily seen that these two types generate the following identical relation.

$$\begin{bmatrix} u \\ v \end{bmatrix} = W := GP = \begin{bmatrix} 1/\sqrt{2b} & \sqrt{b/2} \\ -1/\sqrt{2b} & \sqrt{b/2} \end{bmatrix} \begin{bmatrix} f \\ \dot{x} \end{bmatrix} \qquad (1)$$

where b is the wave impedance which determines the behavior of a system, $\dot{x}$ and f are the velocity and force, u and v are the forward (or incident) and backward (or reflected) waves, respectively. And G represents the transformation matrix, W and P are the vectors of wave variables and power variables, respectively. Since the wave transformation is one-to-one, the inverse of (1) is uniquely determined as

$$\begin{bmatrix} f \\ \dot{x} \end{bmatrix} = P = G^{-1}W = \begin{bmatrix} \sqrt{b/2} & -\sqrt{b/2} \\ 1/\sqrt{2b} & 1/\sqrt{2b} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \qquad (2)$$



(a) Impedance type (b) Admittance type

Fig. 1 Wave transformations

In the practical implementation, the wave transformation for the system that dealt with the power variables must be used as a pair of impedance type and admittance type wave transformation. The transmission through the pair of two wave transformation is called to the *wave transmission*. The wave transmission can be represented in terms of the input and output vectors by the power and wave variables.

$$\begin{bmatrix} f_l \\ -\dot{x}_r \end{bmatrix} = O_P := HI_P = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_l \\ f_r \end{bmatrix} \qquad (3)$$

$$\begin{bmatrix} v_l \\ u_r \end{bmatrix} = O_W := SI_W = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} u_l \\ v_r \end{bmatrix} \qquad (4)$$

where H and S are the hybrid transfer matrix and scattering matrix, h_{ij} and s_{ij} are the components of the hybrid transfer matrix and scattering matrix lying in row i and column j and the subscripts l and r denote the left and right ports, I and O are the input and output vectors and the subscripts P and W denote the power and wave variables, respectively. From the wave transformation, the hybrid transfer matrix of (3) and the scattering matrix of (4) relates as follow:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & b^{-1} \end{bmatrix} \left(\begin{bmatrix} b & 0 \\ 0 & b^{-1} \end{bmatrix} + H \right)^{-1} \left(\begin{bmatrix} b & 0 \\ 0 & b^{-1} \end{bmatrix} - H \right) \begin{bmatrix} 1 & 0 \\ 0 & -b \end{bmatrix} \qquad (5)$$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & -b^{-1} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + S \right)^{-1} \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - S \right) \begin{bmatrix} b & 0 \\ 0 & -1 \end{bmatrix} \qquad (6)$$

From the linear system theory and network theory, the passivity condition of a system can be written as

$$\| S(s) \| \le 1, \quad \forall \omega \qquad (7)$$

That is, if the norm of the scattering matrix is less than or equal to 1, then the system is *passive* [10]. Eq. (7) is equivalent to

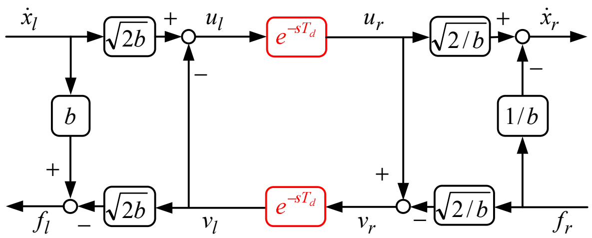


Fig. 2 Time-delayed wave transmission

$$I - S^*(s)S(s) \geq 0, \quad \forall \omega \tag{8}$$

Using Eq. (4), the passivity condition can be rewritten with the components of a scattering matrix as

$$A = |s_{11}|^2 + |s_{21}|^2 \leq 1 \tag{9}$$

$$B = |s_{11}|^2 + |s_{12}|^2 + |s_{21}|^2 + |s_{22}|^2 - |s_{11}s_{12} - s_{21}s_{22}|^2 \leq 1 \tag{10}$$

The wave transformation can be performed in the bilateral teleoperation with time delay. From the viewpoint of passivity, the wave variables are robust to arbitrary time delays [2]. Figure 2 shows a bilateral time-delayed wave transmission. The robustness of wave variables to transmission delay can be easily verified with (7). In the wave space, the input-output relation of wave variables for the time-delayed wave transmission can be written as

$$S(s) = \begin{bmatrix} 0 & e^{-sT_d} \\ e^{-sT_d} & 0 \end{bmatrix} \tag{11}$$

where T_d is the transmission delay. It can be easily known that the time-delayed transmission in the wave space is passive because $\|S(s)\| = 1$.

III. Wave Transformation in the Sampled-data System

As illustrated in Fig. 3, one wave variable u (or v) depends on the information of the other variable v (or u) called the *reflection wave*. More precisely, in computation of $u_l(t)$, $v_l(t-T_s)$ is used because of the time delay T_s occurring in the sampled-data system. Likewise, $u_r(t-T_s)$ is used in computing $v_r(t)$. The term $v_l(t-T_s)$ or $u_r(t-T_s)$ will be called the unit delayed reflection wave or shortly *delayed reflection*.

From Fig.3, the hybrid transfer matrix of the wave transmission through WTDR can be obtained by

$$H(s) = \begin{bmatrix} \dfrac{b(1-e^{-sT_s})^2}{1+e^{-2sT_s}} & \dfrac{2}{1+e^{-2sT_s}} \\ -\dfrac{2}{1+e^{-2sT_s}} & \dfrac{(1-e^{-sT_s})^2}{b(1+e^{-2sT_s})} \end{bmatrix} \tag{12}$$

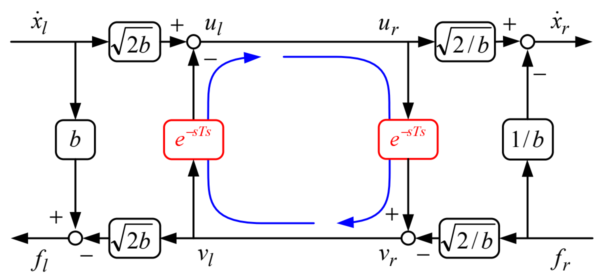


Fig. 3 Wave transmission through WTDR

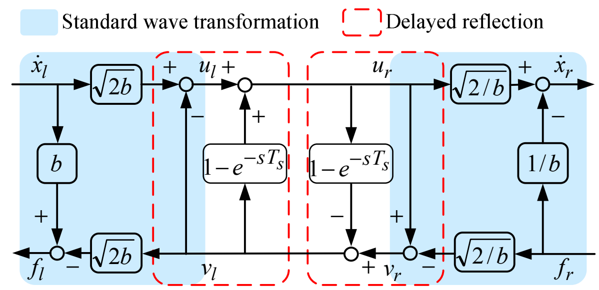


Fig. 4 Schematic of the wave transmission through WTDR

As depicted in Fig. 3, the unit delay which is required for the causality due to the reflection wave makes the wave transformation on the sampled-data system (i.e., WTDR) different from the standard wave transformation on the continuous-time system. The WTDR can be reconfigured by dividing the standard wave transformation and the term of delayed reflection. In Fig. 4, WTDR includes a factor $1-e^{-sT_s}$ which can be represented by a first-order Pade approximation as follows

$$1-e^{-sT_s} \approx \frac{2s}{s+2/T_s} \tag{13}$$

Note that Eq. (14) happens to be kind of a high-pass filter whose gain and cut-off frequency are 3.01dB and $2/T_s$ rad/s, respectively.

From Fig.4 or (5) and (12), in the wave space, the input-output relation of wave variables can be written as

$$S(s) = \begin{bmatrix} -\dfrac{1-e^{-sT_s}}{1+(1-e^{-sT_s})^2} & \dfrac{1}{1+(1-e^{-sT_s})^2} \\ \dfrac{1}{1+(1-e^{-sT_s})^2} & \dfrac{1-e^{-sT_s}}{1+(1-e^{-sT_s})^2} \end{bmatrix} \tag{14}$$

Substitution of Eq. (13) into Eq. (14) yields

$$S(s) = \begin{bmatrix} -\dfrac{2s(s+2T_s^{-1})}{(s+2T_s^{-1})^2+4s^2} & \dfrac{(s+2T_s^{-1})^2}{(s+2T_s^{-1})^2+4s^2} \\ \dfrac{(s+2T_s^{-1})^2}{(s+2T_s^{-1})^2+4s^2} & \dfrac{2s(s+2T_s^{-1})}{(s+2T_s^{-1})^2+4s^2} \end{bmatrix} \tag{15}$$

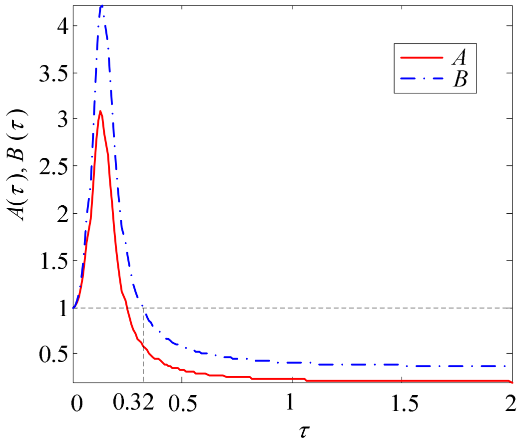


Fig. 5 $A(\tau)$ and $B(\tau)$ of the wave transmission through WTDR

From (15), the functions A and B that verify the passivity of the wave transmission through WTDR can be found as

$$A(\tau) = \frac{1+6(\pi\tau)^2+5(\pi\tau)^4}{1-6(\pi\tau)^2+25(\pi\tau)^4} \quad (16)$$

$$B(\tau) = \frac{1+10(\pi\tau)^2+9(\pi\tau)^4}{1-6(\pi\tau)^2+25(\pi\tau)^4} \quad (17)$$

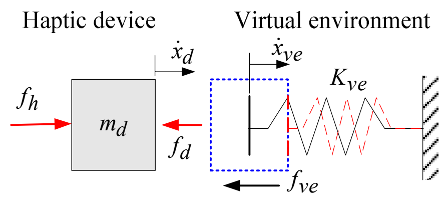
where $\tau = \omega/\omega_s = \omega T_s/(2\pi)$ and ω_s is the sampling frequency. Figure 5 illustrates the curves of $A(\tau)$ and $B(\tau)$ as a function of τ which shows the scattering property of the wave transmission through WTDR. As shown in Fig. 5, the functions $A(\tau)$ and $B(\tau)$ satisfy the passivity condition at either $\tau = 0$ (i.e., at steady state) or $\tau > 0.32$ (i.e., at high frequency). These results agree with the previous analysis based on the hybrid matrix. In conclusion, a system can be made passive by selecting the sampling frequency which satisfies the passivity condition.

IV. Applications to Haptic Interface

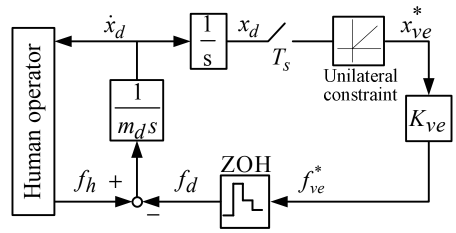
Figure 6 illustrates a simple haptic interface in which the haptic device and virtual environment are modeled by the mass m_d and the spring K_{ve}, respectively. A hand force f_h the human operator applies to the handle of the haptic device causes the device to move toward the virtual environment. When the device is in contact with the wall and compresses it, the reaction force f_{ve} can be computed as the product of the spring stiffness, K_{ve}, and the displacement of the virtual spring, x_{ve}. This reaction force is transmitted to the operator through the haptic device. In practice, the virtual environment cannot be composed of only linear components. The basic haptic interface can be implemented using nonlinear elements such as a sampler, ZOH and unilateral constraint. In Fig.6, the superscript * denotes the discrete-time signal.

The passivity of the sampled-data system can be given by

$$b_d > \frac{T_s}{2}\left(1-\cos\omega T_s\right)^{-1} \mathrm{Re}\{(1-e^{-j\omega T_s})H_{ve}(e^{j\omega T_s})\} \quad (18)$$



a) Schematic



b) Implementation on sampled-data system

Fig. 6 Basic haptic interface.

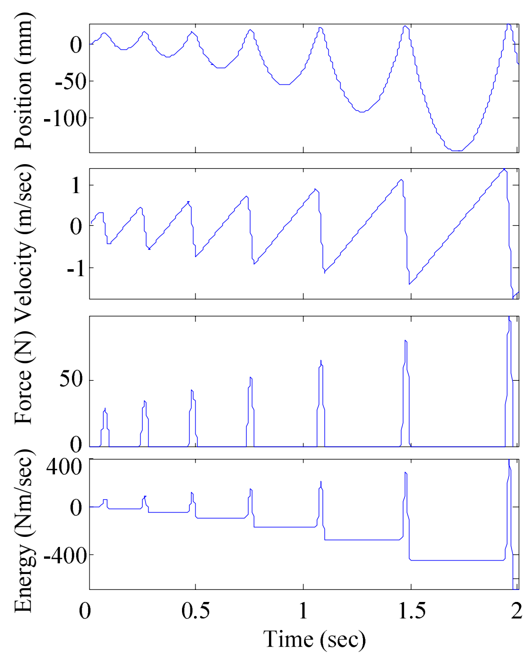


Fig. 7 Example of basic haptic interface (K_{ve}=5kN/m, T_s=1ms, m_d=1kg and f_h=4.5N).

where b_d is the viscous damping of the haptic device, and $H_{ve}(e^{j\omega T_s})$ is the discrete transfer function of the virtual environment [6]. Equation (18) compares the magnitude of viscous damping of the haptic device with the real part of the result from computation of the virtual environment on the sample-data system. As an example, when (18) is applied to the system of Fig. 6, since $b_d = 0$, the system is passive if and only if $K_{ve}T_s < 0$, but it is impossible to satisfy this condition, because $K_{ve} > 0$ and $T_s > 0$. Hence, the system is intrinsically active.

It is well known that a virtual wall can become active while a real physical wall is always passive. Figure 7 illustrates the active nature of a virtual wall by investigating contact of the haptic device with the virtual wall shown in Fig. 6, in which very unstable behavior of the haptic system is observed. Suppose the haptic device initially at 0 mm deformed the wall at 0.01m by the haptic device driven by the hand force. During this operation, the virtual wall modeled as a virtual spring generated energy, thus serving as an active element. The stiffness of the virtual wall K_{ve}, sampling frequency T_s, and hand force f_h are 5kN/m, 1kHz and 4.5N, respectively. This harder contact caused the unstable behaviour, thus leading to the highly oscillatory response and negative energy showing the active nature of the haptic device.

In Section 3, it is known that the wave transmission through WTDR can be a control method for the system that deals with the force and the velocity. The WTDR can be adopted as a force control method for the haptic interface, but little research has been conducted to overcome the problems associated with the active wall of the haptic display. Since the wave transform is intrinsically passive to the transmission delay, it is desirable to design a haptic interface in wave space when transmission delay exists, such that the virtual environment exists on the internet. When the wave transmission through WTDR is applied to the basic haptic interface shown in Fig. 6, the whole system can be represented in Fig. 8.

To verify the passivity of the system, applying the passivity condition of (18) to Fig.8 gives

$$Q(b,\omega)=\frac{2(K_{ve}T-2b)(K_{ve}T+(K_{ve}T-2b)\cos(\omega T))}{4b^2+K_{ve}{}^2T^2+(4b^2-K_{ve}{}^2T^2)\cos(2\omega T)}-1<0 \quad (19)$$

Figure 9 shows the graph for $Q(b, \omega)$ as a function of frequency and wave impedance for K_{ve} = 5kN/m and T_s =1msec, the detail can be shown in [10]. And the passivity ranges are

$$0.333 < b/(K_{ve}T_s) < 3.579 \quad (20)$$

V. Experiments

In the previous section, it was shown from simulations that the haptic interface using wave transformation could be intrinsically passive depending on the choice of the wave impedance. It was assumed in simulations that the haptic device was modeled by mass alone, the human operator had only the output force, and no velocity noise occurred. In the actual systems, however, both the haptic device and the human operator have some impedance composed of a mass, a spring and a damper, and the velocity noise inevitably occurs.

Various experiments have been performed by the PHANToM premium 1.0 from SensAble Technologies [11]. This haptic device is capable of 3D force feedback and 6D position sensing. In the experiments, the virtual wall was located at 10mm with K_{ev} = 5kN/m, and the sampling period was set to 1msec. Figure 10 shows the experimental results of the conventional haptic interface shown in Fig.6. There was no additional haptic controller. The stored energy decreased to a negative value, and the position, velocity and force responses were highly oscillatory, thus indicating the typical unstable behavior of a haptic interface. The magnitudes did not diverge further because of the damping and mass of the human hand.

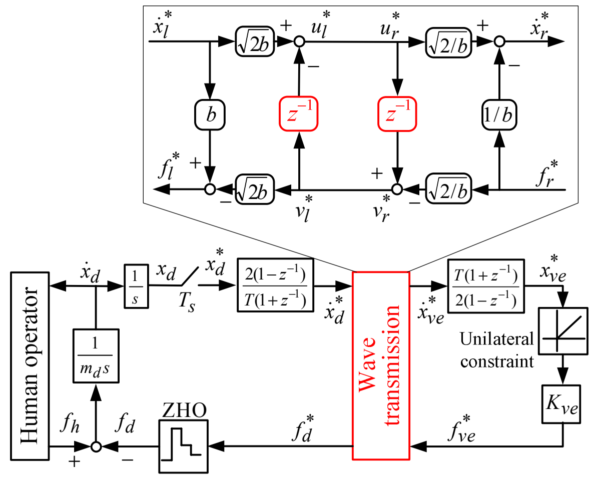


Fig. 8 Haptic interface use of the wave transmission through the WTDR

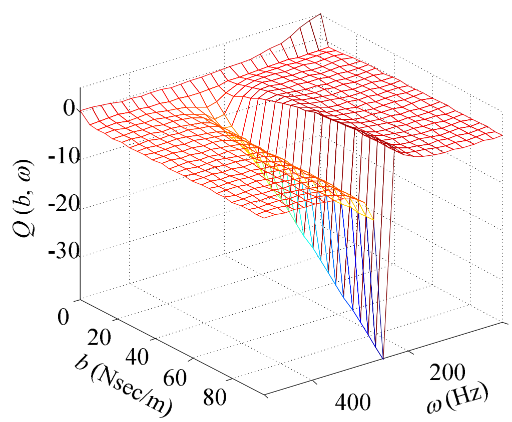


Fig. 9 $Q(b, \omega)$ as a function of frequency and wave impedance (K_{ve} = 5kN/m, T_s = 1msec)

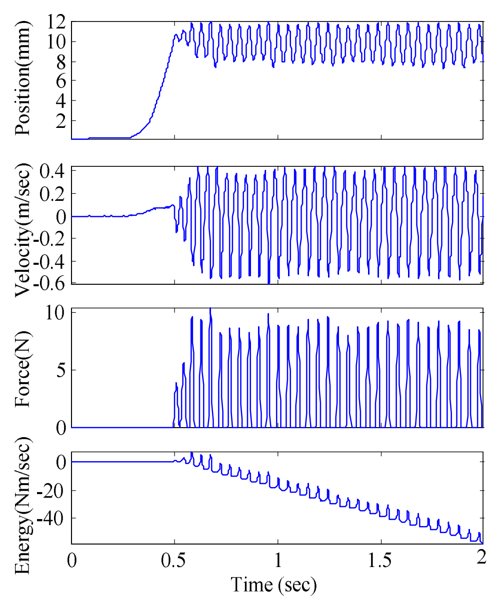


Fig. 10 Experimental results of the conventional haptic interface (K_{ve} = 5kN/m, T_s = 1msec)

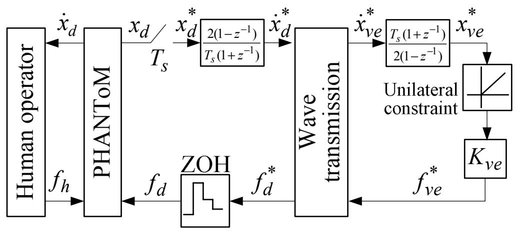


Fig. 11 Haptic interface through wave transformation

Figure 11 shows the schematic of the signal flows of the proposed haptic interface. The haptic device has some backdrive friction and the human operator also has some damping that cannot be modeled accurately. The experimental conditions were the same as those of Fig.10 except for addition of the wave transformation. The wave impedance used in the experiment was 17Nsec/m from (19).

The experimental results of the proposed haptic interface are presented in Fig.12. The result shows the stable behavior of the haptic interface. The position became stabilized rapidly to 10.9mm which was very close to the simulation results, which could be expected from the force and stiffness of the virtual environment. As for the energy of the system, the stored energy of the haptic device remained positive because the device kept contact after the initial contact with the wall. Furthermore, the stored energy of the virtual environment also remained positive. However, when the device returned to initial position, the stored energy of the haptic device remained positive, but the stored energy of the virtual environment became negative. It is noted that the wave variables of the ideal display remained at a constant value after the initial contact, since there was no change in the device velocity and the force from the virtual environment. It is also observed in Fig.12 that the wave variables had the same magnitudes but the opposite sign at the steady state, indicating that the haptic interface was stable. Their magnitude was as $0.76\sqrt{\text{Nm/sec}}$ from Fig.12 or (1).

VI. Conclusions

In this paper, the wave transformation was implemented on the sampled-data system. For this implementation, the unit delayed reflection wave is a necessary component. The wave transmission through *wave transformation using delayed reflection* (WTDR) can be either an active or a passive element according to the sampling period and its property can be verified with the passivity condition of the scattering matrix. From the passivity condition of the sampled-data system, the wave impedance which makes the wave transmission through WTDR serve as positive damping can be found. As an example, the wave transmission through WTDR can make the haptic interface stable on the sampled-data system without additional damping.

The experiments also show that the stable haptic interface can be accomplished by the wave transmission. As a result, it is shown that the wave transmission through WTDR is intrinsically a control law. Therefore, the wave transformation is beneficial not only to the bilateral teleoperation with time delay, but also to the haptic interface implemented on the sampled-data form. Furthermore, the wave transmission through WTDR can serve as a controller for the force controlled system.

References

1. R.J. Anderson and M.W. Spong, Bilateral Control of Teleoperators with Time Delay, *IEEE Trans. on Automatic Control*, 34(5), pp.494-501(1989).
2. G. Niemeyer and J. J. Slotine, Stable Adaptive Teleoperation, *IEEE Journal of Oceanographic Engineering*, 16(1), pp.152-162 (1991).
3. Y. Yokokohji, T. Imaida and T. Yoshikawa, Bilateral Teleoperation under Time-Varying Communication Delay, in *Proc. The IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Kyongju, Korea, pp.1854-1859 (1999).
4. S. Munir and W. J. Book, Internet-Based Teleoperation Using Wave Variables With Prediction, *IEEE/ASME Transactions on Mechatronics*, 7(2), June, pp.124-133 (2002)
5. S. S. Haykin, *Active Netwok Theory*, Addison-Wesley (1970)
6. J. E. Colgate and G. G. Schenkel, Passivity of a Class of Sampled-Data Systems : Application to Haptic Interface, *Journal of Robotic System,* 14(1), pp.37-47 (1997).
7. B. Hannaford and J-H. Ryu, Time-Domain Passivity Control of Haptic Interface, *IEEE Trans. on Robotics and Automation*, 18(1), pp.1-10 (1999).
8. R. Daniel and P. McAree, Fundamental limits of performance for force reflecting teleoperation, *International Journal of Robotics Research* 7(8), pp.811-830 (1998).
9. S. E. Salcudean, M. Zhu, W-H. Zhu and K. Hashtrudi-Zaad, Transparent Bilateral Teleoperation under Position and Rate Control, *IEEE Trans. on Robotics and Automation*, 15(3), pp.465-474 (2000).
10. J-H. Lee, C-H. Cho, J-B. Song, C-S. Hwang, and M. Kim, Haptic Interface using Delayed Reflection Wave: Application to a Passive Haptic Device, in *Proc. The IEEE Int. Conf. on Robotics and Automation*, Barcelona, Spain, (2005).
11. http://www.sensable.com/products/phantom_ghost/premium10.asp

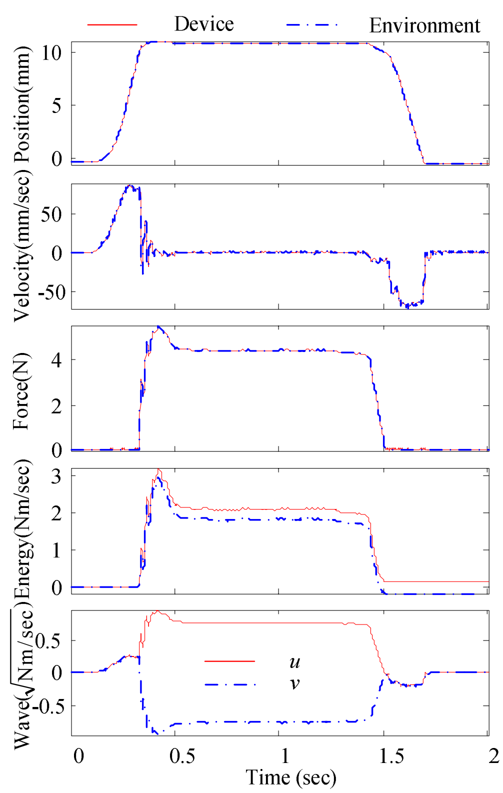


Fig. 12 Experimental results of the proposed haptic interface (K_{ve} = 5kN/m, T_s = 1msec, b = 17Nsec/m)