

# Control of a mobile robot with passive multiple trailers

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**Abstract**— A mobile service robot can achieve reconfigurability by exploiting passive trailer systems. Reconfigurability provides significant practical advantages in order to deal with various service tasks. However, a motion control problem of a passive multiple trailer system is difficult, mainly because a kinematic model is highly nonlinear. It is shown how a robot with  $n$  passive trailers can be controlled in backward direction. Once a desired trajectory of a last trailer is computed, then the control input of a pushing robot is obtained by the proposed control scheme. A kinematic design of a trailer system is proposed in our prior work. It is shown that the high performance of trajectory tracking is also valid for the backward motion control problem. Experimental verifications were carried out with the PSR-2(Public Service Robot) with three passive trailers. Experimental result showed that the backward motion control can be successfully carried out using the proposed control scheme.

**Keywords**- passive trailer, backward motion control, nonlinear kinematic model

## I. INTRODUCTION

Owing to the enormous research achievements in recent robotics technology, service robotic systems are receiving much attention. Many of the service robots have been developed for a specific target application. This fact implies that we need four kinds of the robot for four tasks. The PSR2 is under development at the KIST, towards various indoor service applications, as shown in Figure 1. Major target tasks include patrol, floor cleaning, and luggage transportation in office environments. We have proposed a design scheme of the passive multiple trailer system, which showed excellent trajectory following performance [2]. Exploitation of passive trailers provides reconfigurability which is advantages in practical applications. For example, huge objects can be delivered by a luggage cart. For floor cleaning tasks, a cleaning trailer, which is equipped with a cleanser tank and brush mechanism, can be towed. An operational cost of exploiting a passive trailer system is remarkably lower than operating multiple individual mobile robots. However, it is difficult to control the motion of a multi-body mobile robot system. A kinematic model is represented by highly nonlinear equations. There are two velocity inputs and

$n+3$  generalized coordinates, which implies the trailer system is an underactuated system. Motion control problem is easier when the robot tows passive trailers to the forward direction. A scope of this paper is focused on the backward motion control problem, which is open loop unstable system. Backward motion control provides practical advantages in navigating corridors and a parking problem.



Figure 1. The PSR2 with trailers

There have been many research achievements in design and control of multi-body mobile robotic systems. Laumond[8] showed the controllability of a multiple direct-hooked(or standard) trailer system. Murray and Sastry proposed Chained form in [9], which provided the way to develop many controllers to steer and to stabilize nonholonomic systems, including the multiple trailer system. Typical examples are open loop strategies proposed by Tilbury, Murray and Sastry in [10] and a closed loop controller by Sørtdalen and Wichlund [11]. Rouchon et al. proposed an open loop motion generation strategy using differential flatness [1]. Laumond proposed the virtual robot concept to control the backward motion of one trailer [6]. Altafini proposed the hybrid controller for backward motion of general 3-

trailer [5]. On the other hand, design issues were also studied and some practical examples were proposed by Nakamura et al. [17] and Lee et al [2]. However, it is still difficult to find out practical solutions in order to control a robot with  $n$  passive trailer systems in practical applications. Backward motion control of standard trailer system which can be converted to the chained form is a still difficult problem. Nakamura's system requires a little complicated passive steering mechanism.

In this paper, it is shown how a robot with  $n$  passive trailers can be efficiently controlled in backward direction using one simple controller. The problem is formulated as a trajectory following problem, rather than control of independent generalized coordinates. Once a reference trajectory is given to the  $n$ 'th trailer, then  $n$ 'th trailer is controlled to follow a desired path within an acceptable error range. Then all the other connected trailers successively move along the trajectory of the  $n$ 'th trailer. A kinematic model is iteratively defined between connected adjacent bodies. By using iterative kinematics and trajectory tracking aspects, a control problem can be simplified. From the advantageous kinematic design of the passive trailer systems in [2], it can be shown that the trajectory of  $i$ 'th trailer converges to the trajectory of  $i-1$ 'th trailer. This fact implies that once the desired motion of the  $n$ 'th trailer is obtained, appropriate motion of the pushing mobile robot can be computed easily. A feedback controller can be designed by monitoring the  $n$ 'th trailer's position and joint angles.

Since the proposed controller is developed for general multi-axle robotic systems, many of the mobile robotic system can be controlled by the proposed scheme. For example, a kinematic model of a car-like vehicle corresponds to a mobile robot with one trailer. Therefore, control inputs of such systems can be obtained by appropriate coordinate and input transformation using the kinematics of proposed general multi-body mobile robot. This is another significant advantage of the proposed scheme.

We explain the kinematic design and modeling in Section II. Backward motion controller and the virtual link tracking method (VLTM) are introduced in Section III. VLTM is a simple trajectory tracking controller of a mobile robot. Section IV shows experimental set up of the prototype. Section V describes experimental results. Finally, concluding remarks are presented in Section VI.

## II. KINEMATIC MODELING

Figure 2 illustrates a mobile robot with  $n$  trailer which is proposed in [2]. Let  $x_0, y_0$  be the Cartesian coordinates of the robot current position,  $\theta_0$  the robot orientation with respect to the  $x$ -axis. In this trailer system, the front link of the trailer is hitched on the middle of wheel axis of the robot or trailers. That is, the direct-hooked system is a special case of off-hooked system. However, the properties are different from a control point of view and the systems are studied separately. The robot linear and angular velocities are  $v_0, \omega_0$ , respectively.

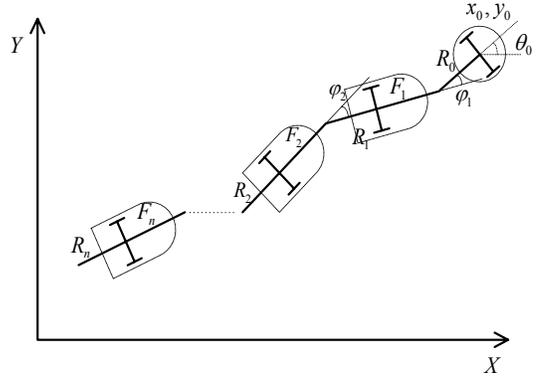


Figure 2. Kinematic model

The kinematic equations of a mobile robot with  $n$  trailers can be written as follows:

$$\begin{aligned}
 \dot{x}_0 &= v_0 \cos \theta_0 \\
 \dot{y}_0 &= v_0 \sin \theta_0 \\
 \dot{\theta}_0 &= \omega_0 \\
 v_1 &= v_0 \cos \varphi_1 + R_0 \dot{\theta}_0 \sin \varphi_1 \\
 \dot{\theta}_1 &= \frac{1}{F_1} \{v_0 \sin \varphi_1 - R_0 \dot{\theta}_0 \cos \varphi_1\} \\
 &\vdots \\
 v_n &= v_{n-1} \cos \varphi_n + R_{n-1} \dot{\theta}_{n-1} \sin \varphi_n \\
 \dot{\theta}_n &= \frac{1}{F_n} \{v_{n-1} \sin \varphi_n - R_{n-1} \dot{\theta}_{n-1} \cos \varphi_n\}
 \end{aligned} \tag{1}$$

where  $\theta_i$  is the  $i$ -th trailer orientation with respect to  $x$ -axis,  $v_i, \dot{\theta}_i$  are linear and angular velocities. We defined  $\varphi_i = \theta_{i-1} - \theta_i$ .  $\varphi_i$  is a relative angle which is measured directly from equipped potentiometers on the hitching point between  $i-1$ 'th and  $i$ 'th trailer. The front link and rear link length are  $F$  and  $R$ , respectively. These constants define the geometry of the robot-trailer connection. And if  $R=0$ , the connection is direct-hooked. When  $R/F=1$ , it is showed that the trajectory tracking error is minimum in [2]. For the simplicity of verification, we assume that the robot navigates with constant velocity ratio  $\lambda=v/\omega$ , then its trajectory will draw a circle. In Figure 3, a robot corresponds to the towing trailer, then  $\lambda=D$ . Then, the relative angle  $\varphi_i$  is obtained as follows:

$$\varphi_i = \begin{cases} 2 \tan^{-1} \left( \frac{D \pm \sqrt{D^2 - (F^2 - R^2)}}{F - R} \right), & F \neq R \\ 2 \tan^{-1} \left( \frac{F}{D} \right), & F = R \end{cases} \quad (2)$$

It is clear that the trajectory tracking error can be minimized if  $R/F=1$ . Therefore, we determined the link parameters of the real trailers to be  $R=F=L$ .

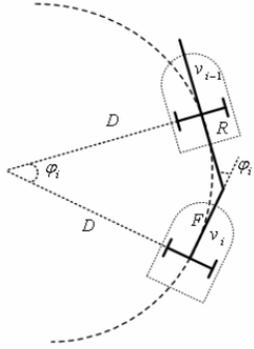


Figure 3. Off-hooked trailer

### III. BACKWARD MOTION CONTROL

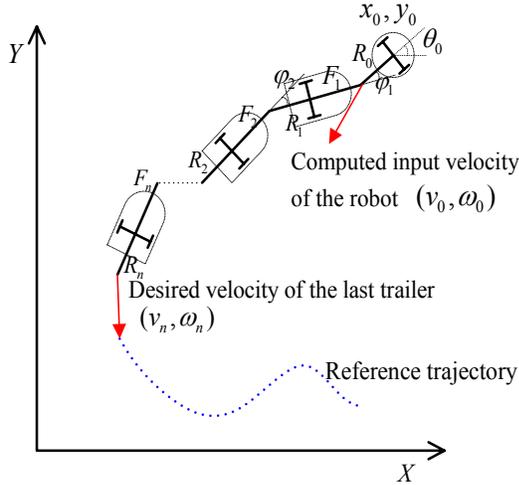


Figure 4. The robot's input velocity is computed by the velocity of the last trailer

Once a desired trajectory of the last trailer is given, then we can calculate the control input of the robot by solving inverse kinematic equations of the trailer system. The desired motion of

the last trailer can be determined by using existing various methods. Inverse kinematics can be represented in equation (3),

$$\begin{aligned} v_0 &= v_n \cos \psi_n + (-1)^{n-1} L \dot{\theta}_n \sin \psi_n \\ \omega_0 &= \dot{\theta}_0 = \frac{1}{L} v_n \sin \psi_n - (-1)^{n-1} \dot{\theta}_n \cos \psi_n \end{aligned} \quad (3)$$

where  $\psi_i = \sum_{k=1}^i (-1)^{k-1} \varphi_k$ .  $\psi_i$  is directly calculated by measured joint angles from potentiometers on the each joint. These equations can be obtained from equation (1).

Since the control input of a towing robot is a linear velocity  $v$  of a hinge point, a proposed control method can be applied to various types of robots which includes both omni-directional and differential drive robot. We propose a simple trajectory tracking method in order to obtain the desired motion of the last trailer.

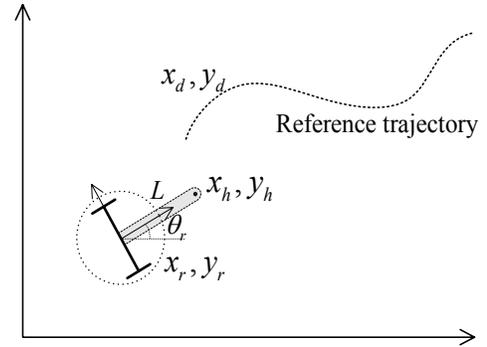


Figure 5. Modeling of a robot to a trailer

There are target points  $x_d, y_d$  which are moving on a reference trajectory, and  $x_r, y_r$  are the current position of a last trailer. Then, we can design a tracking controller using an error position between target points and  $x_h, y_h$ , we suppose that there exist a point set  $x_h, y_h$  which have a constant offset to wheel axis. The method is as following

$$\begin{aligned} \tilde{x}_e &= k(x_d - x_h) \\ \tilde{y}_e &= k(y_d - y_h) \end{aligned} \quad (4)$$

$$\begin{aligned} v_r &= \tilde{x}_e \cos \theta_r + \tilde{y}_e \sin \theta_r \\ \omega_r &= (-\tilde{x}_e \sin \theta_r + \tilde{y}_e \cos \theta_r) / L \end{aligned} \quad (5)$$

where  $k$  is a gain. The controller can be represented by Figure 6.

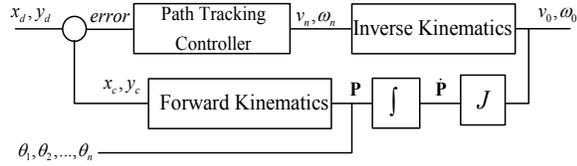


Figure 6. Block diagram of the controller

The robot configuration is  $\mathbf{P} = \{x, y, \theta\}$  and its derivative is  $\dot{\mathbf{P}} = \{\dot{x}, \dot{y}, \dot{\theta}\}$ .

#### IV. PSR2 AND TRAILERS

The PSR2 (see Figure 1) is composed of an omni-directional wheeled mobile platform. Each trailer has two passive wheels and two casters. A wheel orientation is fixed with respect to a trailer body. The distance of a hinge point and the passive wheel axis is equal to that of the axis and a kingpin, 0.52m (Figure 7). The kingpin is using for docking and releasing. The Joint angles are measured using potentiometers which are connected with king-pins by flexible couplers. The coupler can accept the backlash and the misalignment when the trailer moves on the irregular ground condition. The brake system is composed of two solenoids. The brake stops wheels, when the trailer is isolated from a robot. These are controlled remotely through the wireless communication with the robot.



Figure 7. Passive off-hooked trailer (left) and potentiometer (right)

#### V. EXPERIMENT

Experimental verification is carried out for two reference trajectories, which are a straight line and a circle. The PSR2 compute current position of the robot using the laser scanners. The trailers positions are measured from potentiometers. These potentiometers have analog outputs. The link parameters  $R, F$  are 0.52m. The trajectories are predefined on global Cartesian coordinate. Then, we calculate the error position of the last trailer.

Figure 8 shows a resultant experimental trajectory. The PSR2 with 3 trailers is moving in backward direction. A desired reference path is a circle (solid line). Dotted line represents a robot trajectory. A radius of the circular path is 2.5m, however a radius of the robot trajectory is about 2.3m. This is trajectory tracking error, which is acceptably small, due to a performance of the trajectory tracking controller and acceleration conditions. Figure 9 is two control inputs  $v_0$  and  $\omega_0$ , which do not exceed 0.1

m/s and 0.1 rad/s, respectively. These small velocities make a dynamic effect negligible. In a circular path, if the trailer system converges to the path, then relative angles should converge to a constant value. If a radius of the circle is 2.5m, the constant value will be about 25 degree. However, relative angles converge to about 23 degree, in the Figure 10. This error is caused by a trajectory tracking error of the last (fore-going) trailer. The error is acceptably small. It is clear that a robot with trailers can be controlled on a constant curvature path in backward direction.

The reference path is a straight line, in the second experiment. Figure 11 shows the trailer system following the trajectory in backward direction. An initial position of the system is shifted from the desired trajectory; however, the system converges to the desired trajectory, successfully. Same velocity constraints are applied to this experiment. The velocity is shown in the Figure 12. The lower, in the figure, represents angular velocity of the robot. This velocity has to converge to zero, in an ideal case. However, in this experiment, the velocity fluctuates between +0.1 rad/s and -0.1 rad/s, due to potentiometer noise and backlash at the docking joint. It is negligible, because the velocity is small and the mean value is almost about zero. At the last, Figure 14 shows that the joint angles converge to zero, when there is an impulse disturbance that I kicked the 3<sup>rd</sup> trailer. The disturbance is compensated by the controller. Figure 15 shows the velocities of the robot with the disturbance.

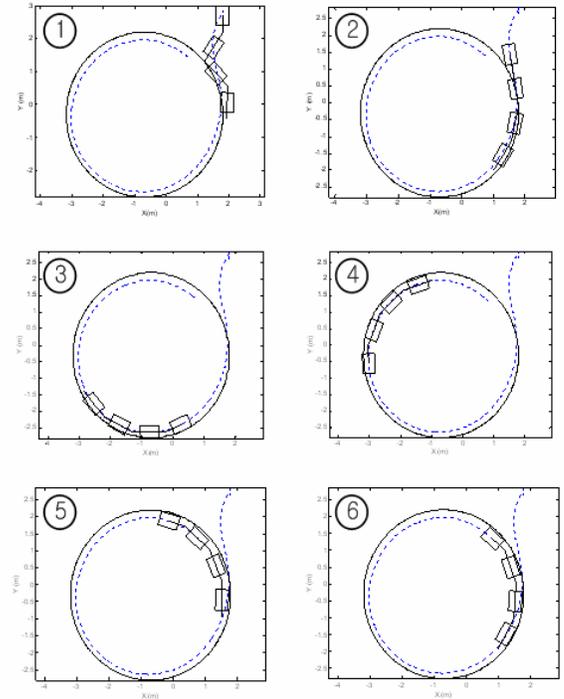


Figure 8. Experimental result : a circle path (radius =2.5 m)

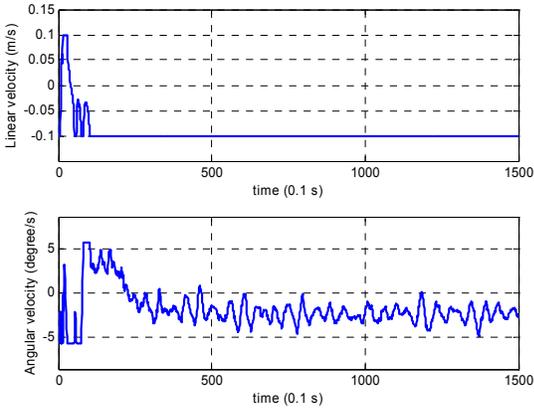


Figure 9. Linear and Angular velocities of the PSR2

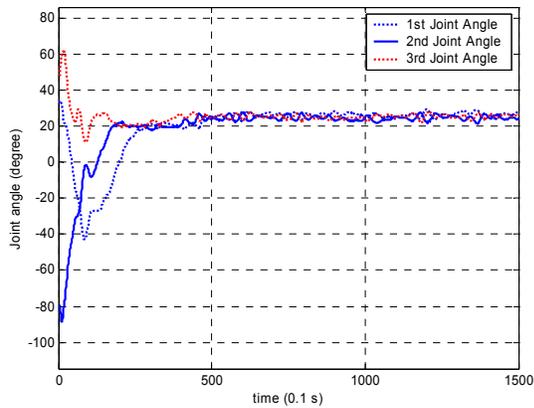


Figure 10. Relative angles between trailers

## VI. CONCLUSION

We proposed a path tracking control method and a backward motion controller for the passive multiple trailer system. It is shown that the system can be controlled successfully. A kinematic parameter design is established, and then backward motion controller is proposed. Since the multi-body system can be controlled to both directions, the system can navigate narrow corridors in practical applications. Furthermore, the proposed controller can be applied to many other classes of multi-axle mobile robotic systems, owing to its generality of kinematics. That is, a multi axle mobile robot with an active driving wheel and an active steering wheel is controllable using a proposed control method. For example, a car-like vehicle can be interpreted as a mobile robot with single trailer (a similar work appears in [15]).

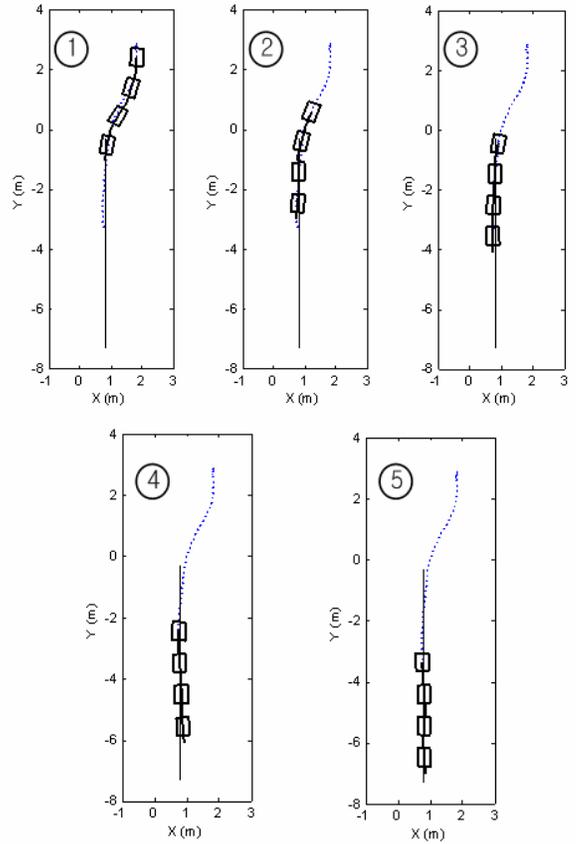


Figure 11. Experimental result : a straight path

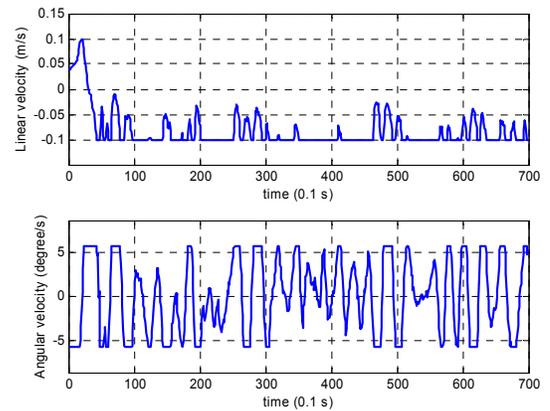


Figure 12. Linear and Angular velocities of the PSR2

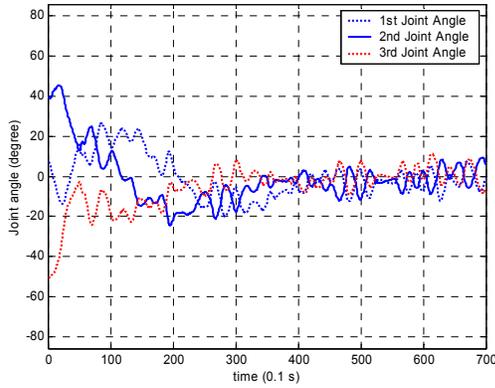


Figure 13. Relative angles between trailers

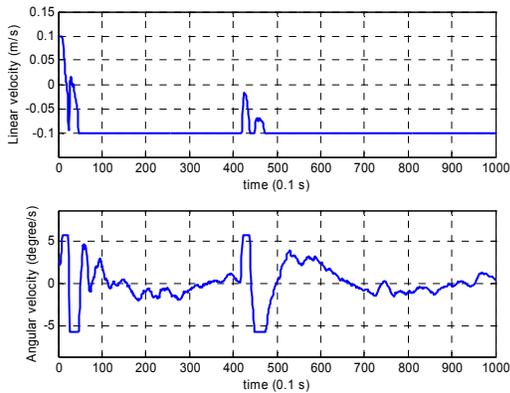


Figure 14. Experimental result (velocities): a straight line path with disturbance

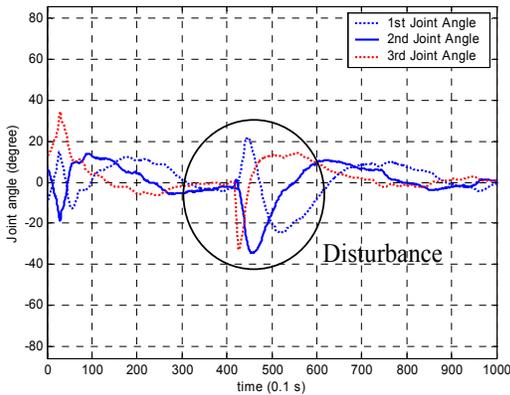


Figure 15. Experimental result (Joint angles): a straight line path with disturbance

## REFERENCES

- [1] P. Rouchon, M. Fliess, J. Levine and P. Martin, "Flatness, motion planning and trailer systems," Proc. of the 32th IEEE Conf. on Decision and Control, pp.2700-2705, San Antonio, Texas, 1993
- [2] Jaehyung Lee, Woojin Chung, Munsang Kim, Chongwon Lee and Jaebok Song. "A passive multiple trailer system for indoor service robots," Proc. of 2001 IEEE/RSJ International Conference on Intelligent Robots and systems, pp. 827-832, Maui, Hawaii, 2001.
- [3] F. Lamiroux, S. Sekhavat and J. P. Laumond, " Motion planning and control for Hilare pulling a trailer," Proc. of 1999 IEEE transaction on robotic and Automation, Vol. 15, NO. 4, pp.640-652, 1999.
- [4] Y. Kanayama, Y. Kimura, F. Miyazaki and T. Noguchi, "A Stable Tracking Control Method for a Non-holonomic Mobile Robot," IEEE/RSJ International Workshop on intelligent Robots and Systems IROS '91, pp. 1236-1241, Osaka, Japan, 1991.
- [5] C. Altafani, A. Speranzon and K. H. Johansson, "Hybrid Control of a Truck Trailer Vehicle," Hybrid Systems V: Computation and Control. LNCS, Springer-Verlag, 2002
- [6] F. Lamiroux and J. P. Laumond, "A practical approach ro feedback control for a mobile robot with trailer," Proc. of the 1998 International Conference on Intelligent Robotics and Automation, pp. 3291-3296, Leuven, Belgium, 1998.
- [7] C. Altafani, "Some properties of the General n-Trailer," International Journal of Control, vol. 74, NO. 4, pp. 409-424, March 2001.
- [8] J. P. Laumond, "Controllability of a Multibody Mobile Robot," IEEE Transactions on Robotics and Automation, VOL. 9, NO. 6, 1993.
- [9] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids", IEEE Transactions on Automatic Control, Vol.38, No.5, pp. 700-716, 1993.
- [10] D. M. Tilbury, R. M. Murray and S. S. Sastry, "Trajectory generation for n-trailer problem using Goursat normal form", Proceedings of the IEEE control and Decision conference, pp. 971-977, 1993.
- [11] O. J. Sordalen, and K. Y. Wichlund, "Exponential stabilization of a car with n trailers", Proceeding of 32nd conference on Decision and Control, pp. 978-983, San Antonio, 1993
- [12] P. Rouchon, M. Fliess, J. Levine and P. Martin, "Flatness and defect of non-linear systems: introductory theory and examples," International Journal of Control, VOL. 61, NO. 6, pp. 1327-1361, 1995.
- [13] J. P. Laumond, "Robot Motion Planning and Control," Lecture notes in control and information sciences, Springer-Verlag, 1998.
- [14] J. Craig, "Introduction to robotics: mechanics and control," 2<sup>nd</sup> edition, Addison-Wesley Publishing Company, 1989.
- [15] S. Sekhavat, J. Hermosillo, "The Cycab Robot : a Differentially Flat System," Proc. Of the 2000 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 312-317, 2000.
- [16] J. A. Reeds and L. A. Shepp, "Optima paths for a car that goes both forwards and backwards," Pacific Journal of Mathematics, Vol. 145. No. 2, 1990.
- [17] Y. Nakamura, H. Ezaki and W. J. Chung, "Design of steering mechanism and control of nonholonomic trailer system," Jounal of robotics Society of Japan, pp. 839-847, 1999.