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Quaternion-Based Orientation Estimation with Static Error Reduction

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***Abstract*— Tracking the orientation or attitude of an object has many applications in various research fields. This paper presents a tracking system which estimates the orientation of an object using an angular rate sensor, accelerometer, and magnetometer. For the combined usage of the sensors, the Kalman filter and the Factored Quaternion Algorithm (FQA) are applied to the proposed system. We also propose a method to eliminate the drift effect of the angular rate sensor in static state. The experimental results show that the proposed tracking system can estimate an accurate rotation of an object and can eliminate the drift effect in static state.**

I. Introduction

Real-time tracking an object is one of the important issues in the field of robotics. Generally, tracking is defined as the observation of a moving object and the supply of the timely ordered sequence of the respective location data of a model. The tracking system is categorized into two types: the optical tracking system and the inertial and magnetic tracking system. Optical tracking system determines the position and the orientation of an object by using multiple cameras and markers. This system measures position and orientation of an object from the relative movement of markers. Some markerless tracking approaches using the optical sensors are also proposed in recent years. However, the optical sensors are fixed outside of an object to observe the object. Therefore, the optical tracking system has the range limitation and cannot observe the object hidden by obstacles. Another category of the object tracking system is the inertial and magnetic tracking system using the Inertial Measurement Unit (IMU) and the magnetometer. IMU is an electronic device that measures the angular rate, orientation, and the acceleration of gravity using the combination of an angular rate sensor and accelerometer. A magnetometer is an instrument to measure the magnetic field in nature. IMU and the magnetometer are small in size and lightweight, and it is possible to build with low cost. When tracking an object, IMU and the magnetometer have no range limitations and are free from obstacles. These sensors can measure the orientation, angular rate, and acceleration of an object in real-time. The tracking system proposed in this paper also uses in an angular rate sensor, accelerometer, and a magnetometer. For the combined use of an angular rate sensor, accelerometer, and magnetometer, the Kalman filter and the Factored Quaternion Algorithm (FQA)[3] are applied to the proposed system. Kalman filter is used to estimate accurate orientation in real-time[1-5][9-10][12], and the FQA is used to improve computational efficiency and avoid singularity. We also propose the additional algorithm to minimize the estimation error of the Kalman filter in static state. This additional algorithm helps to eliminate the drift error derived from the angular rate sensor. Finally, we show the effectiveness of the proposed system with some experimental results.

TABLE I
Sensor Specifications

	Model (Manufacturer)	Range	axis
Angular rate sensor	IDG-300 (InvenSense)	±500°/sec	x,y
	LPY530AL (STMicroelectronics)	±300°/sec	x,z
Accelerometer	MMA7260QT (Freescale Semiconductor)	±6g	x,y,z
Magnetometer	AMI302 (AICHI STEEL)	±0.2mT	x,y,z

II. Sensor Module Design

In this section, a design of sensor module containing an angular rate sensor, accelerometer, and magnetometer is introduced. The specifications of the sensors used in this paper is shown in table I. The goal of the proposed tracking system is to capture human motion, and the sensor specifications are selected to satisfy the human motion. Based on the previous research[6-13], the accelerometer has the range within ±5g, and the angular rate sensor has the range within ±300°/sec, for human motion.

The output of the sensors is analog signal and is converted into digital signal by 12 bit resolution analog to digital converter (ADC). Micro Control Unit (MCU), Texas Instruments Stellaris LM3S808, is used for arithmetic. Fig. 1 shows the block diagram of the sensor module. The sensors are connected with ADC, and the MCU attains the digital sensor signal from the ADC and calibrates it. Fig. 2 is the sensor module for the experiment.

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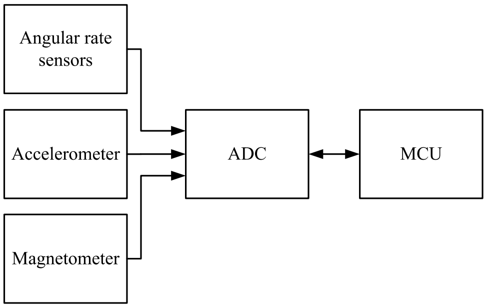

Fig. 1. Block diagram of the sensor module

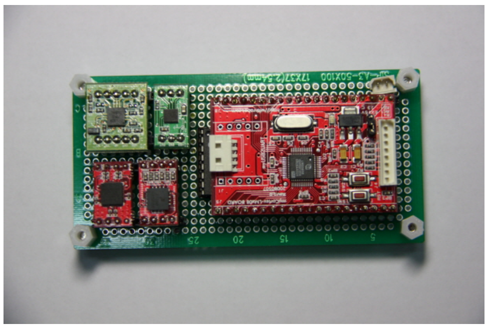

Fig. 2. The sensor module for the experiment

III. Factored Quaternion Algorithm(FQA)

The combination of an accelerometer and magnetometer enables the estimation of the orientation of an object. The acceleration of gravity and magnetic field can be the reference frame, since they are unique in static state. However this is not true in dynamic situation, because the accelerometer measures not only the acceleration of gravity but also the linear acceleration of object motion. In order to overcome this problem, an optimal estimation with the angular rate sensor is needed, which will be explained later. In this section, a measurement model with an accelerometer and magnetometer is introduced to estimate the orientation.

The Factored Quaternion Algorithm (FQA)[3] is one of the algorithms for estimating the orientation based on the acceleration of gravity and magnetic field. The FQA can represent angles in quaternion and can decouple accelerometer and magnetometer data. This decoupling eliminates influence of each other. The following explains the FQA.

The reference coordinate system $\{x_{ref}, y_{ref}, z_{ref}\}$ is defined as follows: ENU (East, North, Up) convention, where x_{ref} points East, y_{ref} points North, and z_{ref} points up. The orientation in this algorithm will be calculated by z,(ψ, azimuth or yaw rotation), y(θ, elevation or pitch rotation), and x(ϕ, bank or roll rotation) axes in sequence. In no rotation of an object, the acceleration of gravity is measured as $-g$ of on z-axis.

The quaternion has four parameters, and forms as follows:

$$q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (1)$$

The quaternion is composed of a scalar part,$\{q_0\}$, and a vector part,$\{q_1, q_2, q_3\}$, where $q_0^2+q_1^2+q_2^2+q_3^2 = 1$. The final orientation in this paper is represented in quaternion.

A. Elevation Quaternion

If an object is rotated by pitch angle θ about the y-axis, the acceleration along the x-axis and z-axis are

$$a_x = g\sin\theta \quad (2)$$
$$a_z = -g\cos\theta \quad (3)$$

where $g = 9.81$ m/s^2 is the acceleration of gravity, and $a = [a_x \ a_y \ a_z]^T$ is the measured acceleration vector in the sensor coordinate system attached to the sensor. The normalized acceleration vector is

$$\bar{a} = \frac{a}{|a|} = \begin{bmatrix} \bar{a}_x \\ \bar{a}_y \\ \bar{a}_z \end{bmatrix} \quad (4)$$

where $|a|$ is the norm of the acceleration vector a, and

$$\sin\theta = \bar{a}_x \quad (5)$$
$$\cos\theta = \sqrt{1-\sin^2\theta} \quad (6)$$

For the positive value of $\cos\theta$, the range of θ is restricted to $-\pi/2 \le \theta \le \pi/2$. In order to obtain the elevation quaternion, half-angle values are as follows

$$\sin\frac{\theta}{2} = \mathrm{sgn}(\sin\theta)\sqrt{(1-\cos\theta)/2} \quad (7)$$
$$\cos\frac{\theta}{2} = \sqrt{(1+\cos\theta)/2} \quad (8)$$

where the sign function is defined as

$$sgn(x) = \begin{cases} -1 & \text{if} \quad x<0, \\ 0 & \text{if} \quad x=0, \\ 1 & \text{if} \quad x>0. \end{cases} \quad (9)$$

The elevation quaternion is represented as follows

$$q_e = \cos\frac{\theta}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \sin\frac{\theta}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (10)$$

B. Roll Quaternion

If the object is rotated about the roll angle ϕ after rotating pitch angle θ, the acceleration of the x-axis is (2) and the acceleration of the y and z axes become

$$a_y = -g\cos\theta\sin\phi \tag{11}$$
$$a_z = -g\cos\theta\cos\phi \tag{12}$$

After applying the normalization, the accelerations become

$$\bar{a}_y = -\cos\theta\sin\phi \tag{13}$$
$$\bar{a}_z = -\cos\theta\cos\phi \tag{14}$$

values of $\sin\phi$ and $\cos\phi$ are

$$\sin\phi = -\bar{a}_y/\cos\theta \tag{15}$$
$$\cos\phi = -\bar{a}_z/\cos\theta \tag{16}$$

The range of ϕ is restricted to $-\pi < \phi \le \pi$, and if $\cos\theta$ is equal to zero, $\sin\phi$ and $\cos\phi$ are not determined. Thus, the preprocessing method for singular avoidance is needed[3]. In similar manner of (7) and (8), the roll quaternion is represented as follows

$$q_r = \cos\frac{\phi}{2}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + \sin\frac{\phi}{2}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} \tag{17}$$

C. Azimuth Quaternion

The azimuth quaternion is calculated lastly. Comparing with the elevation and roll quaternion which use acceleration data, the azimuth quaternion is decoupled from the others, and uses the magnetic field data after operating the elevation and roll quaternion. Let $m = [m_x\ m_y\ m_z]^T$ be the normalized magnetic field vector which is measured in the sensor coordinate system.

${}^a m$ is the normalized magnetic field vector that has no effect of elevation and roll quaternion by the following relation.

$${}^a m = q_e\ q_r\ m\ q_r^{-1}\ q_e^{-1} \tag{18}$$

The relation of ${}^a m$, the vector rotated by the azimuth angle, and the north direction positive y-axis, $[0\ 1]^T$ of the reference frame, is as follows:

$$\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}{}^a m_x\\ {}^a m_y\end{bmatrix} \tag{19}$$

Note that the value along the z-axis is not considered here, because ${}^a m_z$ is coincident with the z-axis of the reference frame. Note also that ${}^a m_x$ and ${}^a m_y$ with ${}^a m_x^2 + {}^a m_y^2 = 1$, are normalized for rotation about the z-axis. The value of $\cos\psi$ and $\sin\psi$ are obtained from (19) as follows:

$$\begin{bmatrix}\cos\psi\\ \sin\psi\end{bmatrix} = \begin{bmatrix}{}^a m_x & {}^a m_y\\ -{}^a m_y & {}^a m_x\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} \tag{20}$$

The range of ψ is restricted to $-\pi < \psi \le \pi$, and by the calculation in similar manner of (7) and (8), the azimuth quaternion is represented as follows:

$$q_a = \cos\frac{\psi}{2}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + \sin\frac{\psi}{2}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \tag{21}$$

The final quaternion, $\hat{q}$, for the orientation with the roll, elevation, and azimuth quaternions of the object are calculated as follows:

$$\hat{q} = q_a\ q_e\ q_r \tag{22}$$

D. Singularity Avoidance

When $\cos\theta = 0$ in (15) and (16), the singularity occurs. So, it should be preprocessed for avoiding singularity[3].

If $\cos\theta$ is close to 0, the normalized acceleration vector and the normalized magnetic field vector are rotated on y-aixs by offset angle α which is set to avoiding $\cos\theta$ as 0.

$$\bar{a}_{\text{off}} = q_\alpha\ \bar{a}\ q_\alpha^{-1} \tag{23}$$
$$m_{\text{off}} = q_\alpha\ m\ q_\alpha^{-1} \tag{24}$$

where $q_\alpha = [\cos(\alpha/2)\ 0\ \sin(\alpha/2)\ 0]^T$. And then, after obtaining the resultant quaternion, $\hat{q}_{\text{off}}$, with q_α, we get final quaternion, $\hat{q}$, as follows:

$$\hat{q} = \hat{q}_{\text{off}}\ q_\alpha \tag{25}$$

IV. Optimal Estimation with Static Error Reduction

In order to estimate the orientation of an object, two models are used: one is the FQA with the accelerometer and magnetometer, and the other with the angular rate sensor. The orientation from the FQA is not suitable in dynamic motion, because the accelerometer measures the acceleration of gravity and motion simultaneously. The orientation with angular rate sensor estimates the orientation relative to the initial orientation, and the error is accumulated in the process of integration.

Thus, for emphasizing each of the orientation from the FQA and from the integrated angular rate, data fusion for optimization is needed[15].

A. Kalman Fitler Process Model

The relation between the quaternion derivative, $\dot{q}$, and the angular rate, $\omega = [\omega_1\ \omega_2\ \omega_3]^T$, measured by the angular rate sensor is as follows[14]:

$$\dot{q} = \frac{1}{2} q \otimes \omega \tag{26}$$

where $\otimes$ is the quaternion multiplication. To obtain the quaternion q, the quaternion derivative, $\dot{q}$, needs to be integrated, and the resultant quaternion should be normalized to

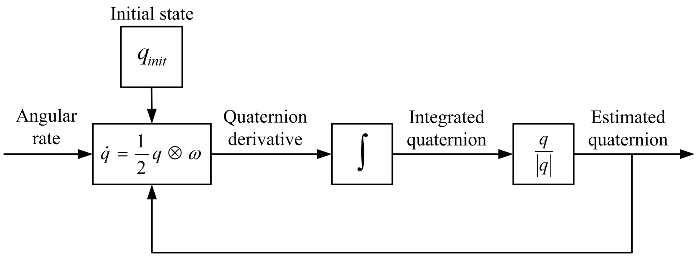

Fig. 3. Block diagram of the process for the orientation represented in quaternion

represent rotational relation. Fig. 3 shows the block diagram of the process for the orientation represented in quaternion, and is used for the process model in Kalman filter.

B. Kalman Fitler

The orientation with the integrated angular rate in the previous section, can be redesigned by using the Kalman filter as follows. The state vector is $q = [x_1\ x_2\ x_3\ x_4]^T$, and the relation of (26) becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{27}$$

The measurement data used in the Kalman filter is equal to the result of the FQA algorithm estimating the orientation using the accelerometer and magnetometer.

$$z = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{28}$$

where z is the measurement for the Kalman filter and $v = [v_1\ v_2\ v_3\ v_4\ ]^T$ is the measurement noise assumed to be white, zero-mean, uncorrelated.

The state equation is

$$\dot{x} = f(x) + w_n \tag{29}$$

where w_n is the process noise and is equal to zero vector, $w_n = [w_{n1}\ w_{n2}\ w_{n3}\ w_{n4}]^T = 0$. The discrete-time model of (29) is

$$x_k = F_k x_{k-1} \tag{30}$$

where the transition matrix is

$$F_k = \begin{bmatrix} 1 & -\frac{\omega_1 \Delta t}{2} & -\frac{\omega_2 \Delta t}{2} & -\frac{\omega_3 \Delta t}{2} \\ \frac{\omega_1 \Delta t}{2} & 1 & \frac{\omega_3 \Delta t}{2} & -\frac{\omega_2 \Delta t}{2} \\ \frac{\omega_2 \Delta t}{2} & -\frac{\omega_3 \Delta t}{2} & 1 & \frac{\omega_1 \Delta t}{2} \\ \frac{\omega_3 \Delta t}{2} & \frac{\omega_2 \Delta t}{2} & -\frac{\omega_1 \Delta t}{2} & 1 \end{bmatrix} \tag{31}$$

and the discrete measurement model is

$$z_k = H_k x_k + v_k \tag{32}$$

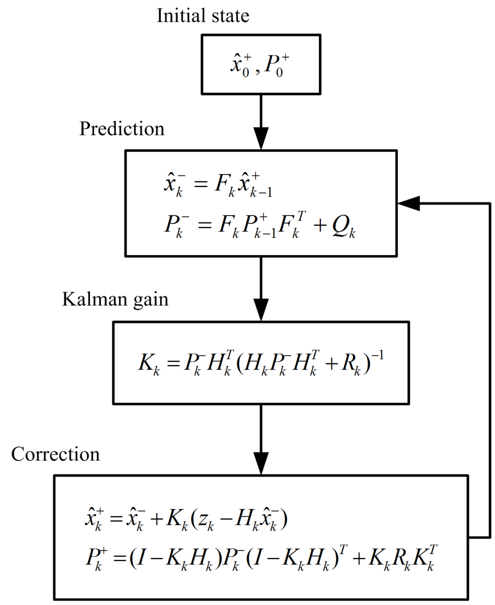

Fig. 4. Block diagram of the process of the Kalman filter

where H_k is the 4×4 Identity matrix, and v_k is the discrete measurement noise.

The process noise covariance matrix for the Kalman filter is

$$Q_k = E[w_k w_k^T] \tag{33}$$

where E is the expectation operation, and w_k is the process noise which is zero, and thus Q_k is the 4×4 zero matrix.

The measurement noise covariance matrix for the Kalman filter is

$$R_k = E[v_k v_k^T] \tag{34}$$

R_k is the 4×4 diagonal matrix, and the elements are the variance determined measuring the quaternion of z.

The process of the Kalman filter is shown in Fig. 4. The initial state is needed at starting the Kalman filter. And the Kalman filter is processed with prediction and correction. P is the covariance of the state, and K is the Kalman gain. The signs changing from '$-$' to '$+$' is the update of the state from prediction to correction.

In order to see the effect of each method, an experiment is performed in static state. Fig. 5. shows the results of the integration of the angular rate, the FQA with accelerometer and magnetometer, and the estimation from the Kalman filter. The orientation of integrated angular rate tends to drift due to the gyro drift. The Kalman filter eliminates the drift error, but not completely. Thus, for eliminating the drift error, the modified estimation is introduced.

C. Error Reduction in Static State

The orientation of integrated angular rate is suitable for measuring the orientation in dynamic state, but not in static state, due to the drift error. The orientation with the FQA is

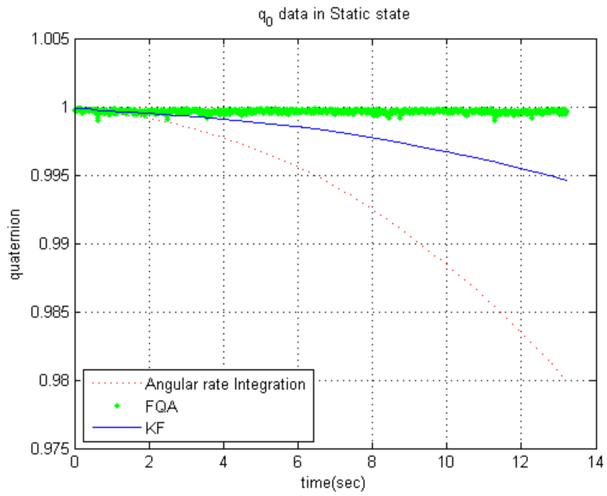

Fig. 5. q_0 in static state with the orientation by integrating the angular rate, measured from the FQA, and estimated from the KF

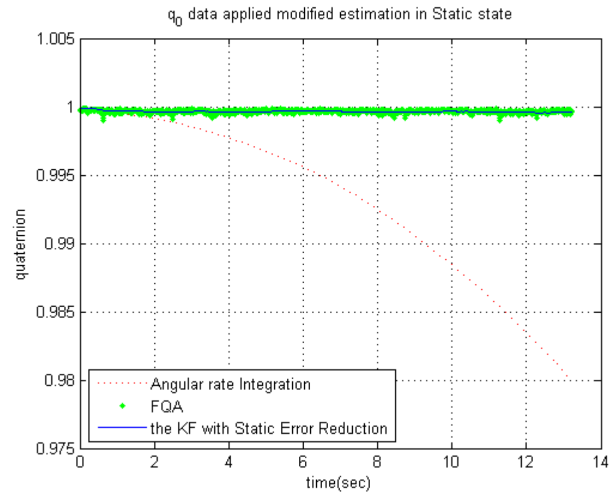

Fig. 7. q_0 in static state with the orientation by integrating the angular rate, measured from the FQA, and modified estimation from the KF

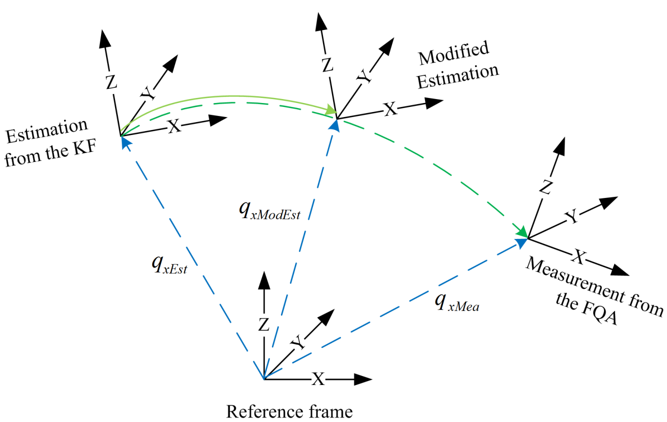

Fig. 6. The orientation of the modified estimation

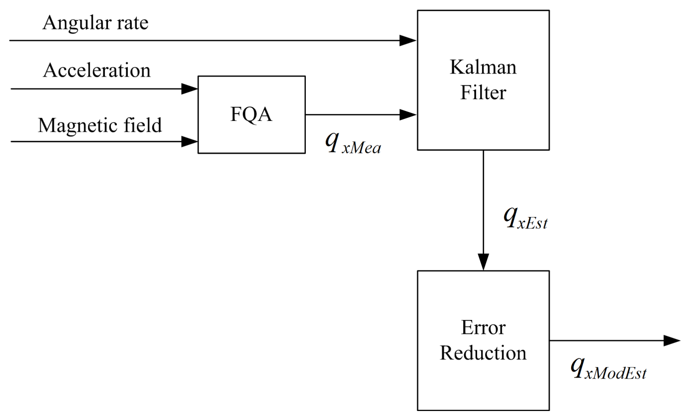

Fig. 8. the block diagram of the design for estimating the orientation

suitable for measuring in static state, but not in dynamic state, because of the linear acceleration by motion. Thus, when the static state occurs, the orientation estimated from the Kalman filter is rotated toward the orientation with the FQA.

Fig. 6 shows the modification of the orientation, where q_{xEst} is the orientation estimated from the Kalman filter and q_{xMea} is the orientation with the FQA. For obtaining the orientation between the orientation estimated from the Kalman fitler and the orientation measured from the FQA, quaternion linear interpolation is used. Quaternion linear interpolation (Lerp) is the linear interpolation between two quaternions by a straight line. Due to the interpolation by straight line, the magnitude of the interpolated quaternion changes. Thus, the normalization of the interpolated quaternion is needed. The orientation of the modified estimation by Lerp is

$$q_{xModEst} = (1 - G_{intp})q_{xEst} + G_{intp}q_{xMea} \qquad (35)$$

where G_{intp} has the range of 0 to 1, and the interpolated position of the quaternion is decided by the value. If G_{intp} is equal to 0, $q_{xModEst} = q_{xEst}$, and if G_{intp} is equal to 1, $q_{xModEst} = q_{xMea}$. For the usage of the orientation quaternion, $q_{xModEst}$ should be normalized.

The experiment of the modified estimation of the orientation is performed in static state. Fig. 7 shows the data of the integration of angular rate, measurement from the FQA, and modified estimation from the Kalman filter. Notice that unlike the graph from Fig. 5, the drift effect is eliminated. The block diagram of the design for estimating the orientation is shown in Fig. 8.

V. Experiment

The modified estimation is applied only in the static state. For distinguishing the state between the static state and dynamic state, the acceleration from the accelerometer is used. If the acceleration from the accelerometer indicates $9.8m/sec^2$, the static state is expected. In this implementation, if the range of $9.8m/sec^2 \pm \alpha$ is measured from the accelerometer, the static state is expected($\alpha = 1m/sec^2$ in this experiment). The experiment is conducted so that the sensor is rotated along the x-axis by 90deg, and then, rotated back to the initial orientation. Fig. 9 shows the orientation with the Kalman

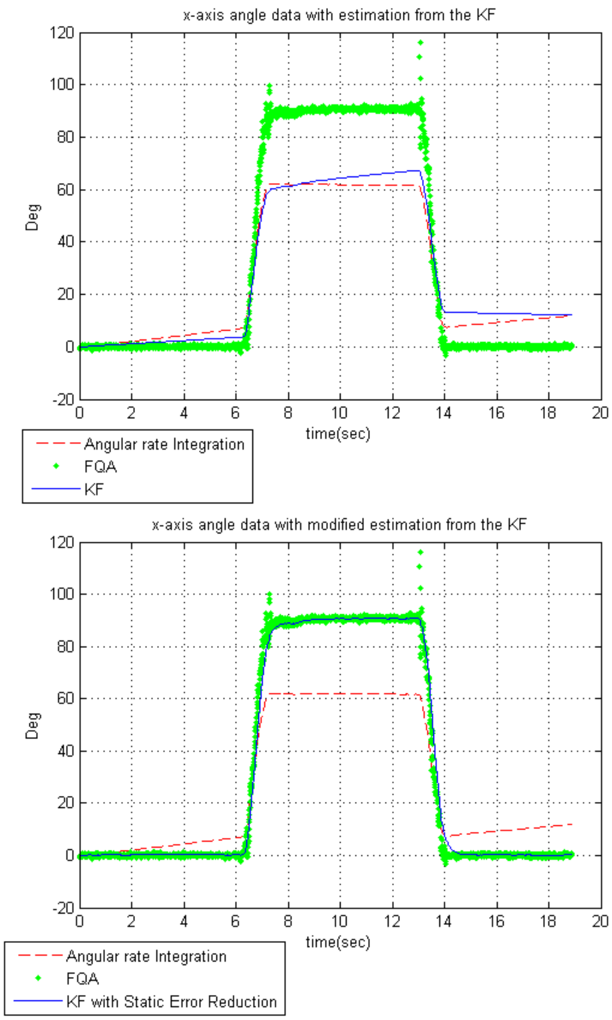

Fig. 9. x-axis angle data rotating x-axis by 90deg with the orientation by integrating the angular rate, measured from the FQA, and the orientation with the Kalman filter (first plot) and the orientation with the Modified Estimation(second plot)

filter, the orientation with modified estimation, the orientation of integrated angular rate and the orientation with the FQA. The orientation with the Kalman filter is effected by the drift error, but the orientation from the modified estimation tends to follow the orientation from the FQA in the static state, eliminating the drift effect.

VI. Conclusion

This paper proposes the method to estimate the orientation of an object by using an angular rate sensor, accelerometer, and magnetometer. To present the orientation, the FQA and Kalman filter are applied in this paper, the Kalman filter is used to estimate the orientation in real-time and the FQA is used to improve the computational efficiency and to avoid singularity. To minimize the drift error in static state, additional algorithm is designed to modify the output of the Kalman filter. The experimental result shows the performance of the proposed tracking system. The drift error in the static state is reduced significantly proving that the system can estimate the orientation of the object.

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References

[1] X. Yun and E. R. Bachmann, "Design, Implementation, and Experimental Results of a Quaternion-Based Kalman Filter for Human Body Motion Tracking", IEEE Transactions on Robotics, Vol. 22, No. 6, Dec. 2006.
[2] X. Yun, C. Aparicio, E. R. Bachmann, and R. B. McGhee, "Implementation and Experimental Results of a Quaternion-Based Kalman Filter for Human Body Motion Tracking", Proceedings of the 2005 IEEE International Conference on Robotics and Automation, pp.317-322., Apr. 2005.
[3] X. Yun, E. R. Bachmann, and R. B. McGhee,"A Simplified Quaternion-Based Algorithm for Orientation Estimation From Earth Gravity and Magnetic Field Measurements", IEEE Transactions on Instrumentation and Measurement, Vol. 57, No. 3, Mar. 2008.
[4] C. Aparicio, "IMPLEMENTATION OF A QUATERNION-BASED KALMAN FILTER FOR HUMAN BODY MOTION TRACKING USING MARG SENSORS", thesis, Naval Postgraduate School, Monterey, CA, Sep. 2004.
[5] T. Harada, T. Gyota, Y. Kuniyoshi, and T. Sato, "Development of Wireless Networked Tiny Orientation Device for Wearable Motion Capture and Measurement of Walking Around, Walking Up and Down, and Jumping Tasks", Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 4135-4140, Oct. 29 - Nov. 2, 2007.
[6] T.Liu, Y.Inoue, K. Shibata, "A Wearable Force Plate System to Successively Measure Multi-axial Ground Reaction Force for Gait Analysis", IEEE International Conference on Robotics and Biomimetics, Dec. 2009, pp. 836-841.
[7] Q. Li, J. A. Stankovic, M. A. Hanson, A. T. Barth, J. Lach, and G. Zhou, Accurate, Fast Fall Detection Using Gyroscopes and Accelerometer-Derived Posture Information", Sixth International Workshop on Wearable and Implantable Body Sensor Networks, 2009, pp. 138-143.
[8] K. Liu, T. Liu, K. Shibata, and Y. Inoue, "Ambulatory measurement and analysis of the lower limb 3D posture using wearable sensor system", International Conference on Mechatronics and Automation, Aug. 2009, pp 3065-3069.
[9] N. Abbate, A. Basile, C. Brigante, and A. Faulisi, "Development of a MEMS based wearable motion capture system", 2nd Conference on Human System Interactions, 2009, pp. 255-259.
[10] Z. Zhang, Z. Wu, J. Chen, and J. Wu, "Ubiquitous human body motion capture using micro-sensors", IEEE International Conference on Pervasive Computing and Communications, 2009, pp. 1-5.
[11] S. Sessa, M. Zecca, Z. Lin, T. Sasaki, K. Itoh, and A. Takanishi, "Waseda Bioinstrumentation System #3 as a tool for objective rehabilitation measurement and assessment - Development of the inertial measurement unit -, IEEE International Conference on Rehabilitation Robotics, Jun. 2009, pp. 115-120.
[12] Y. Fujimori, Y. Ohmura, T. Harada, and Y. Kuniyoshi, "Wearable motion capture suit with full-body tactile sensors", IEEE International Conference on Robotics and Automation, May. 2009, pp. 3186-3193.
[13] T. Liu, Y. Inoue, K. Shibata, "Development of a wearable sensor system for quantitative gait analysis", Measurement, Vol. 42, Iss. 7, Aug, 2009, pp. 978-988.
[14] J. B. Kuipers, "Quaternions and Rotation Sequences", Princeton, NJ:Princeton Univ. Press, 1999.
[15] D. Simon, "Optimal State Estimation", John Wiley & Sons, 2006.